A High-Order Elliptic PDE Based Level Set Reinitialisation Method Using a Discontinuous Galerkin Discretisation With Applications to Topology Optimisation

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Abstract

Level set reinitialisation methods are a group of methods which allow one, at any iteration during the solution of a level set evolution problem, to rebuild the level set function such that it becomes a close approximation to a signed distance function, which is often necessary to ensure stability. Reinitialisation is considered a necessary evil however, as it both increases the computational expense of the problem and can reduce the accuracy of the parent method through shifts in the position of the zero isocontour of the level set function. It is the aim of this work to advance the level set methodology through the adoption of a discontinuous Galerkin (DG) discretisation. DG methods have a number of advantages when compared with continuous Galerkin (CG) finite elements, including trivial implementation of parallelisation, and *hp*-adaptivity, which means that one can improve the time requirements for expensive problems as well as achieving high-order accuracy, which is particularly desirable in the context of a level set reinitialisation method, as it may suffice to remedy some of the previously stated issues. A number of the preferred methods of reinitialisation, do not trivially translate to DG. For example, see previous work on the geometric method [1], fast marching methods [2], fast sweeping methods [3], and the Hyperbolic PDE based reinitialisation method [4]. Where these methods have been applied successfully to discontinuous problems, they have often not been shown to achieve the desired high-order accuracy. In this work we present a fully DG level set method, with emphasis on an optimally convergent level set reinitialisation method based on the solution of an elliptic PDE, a CG solution to which is presented in [5].

References

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