Computation of Distributions of Samples Taken at Random Events

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Abstract

Analyzing statistical distributions of variables observed at random events of a stochastic process or field is important in many practical problems. As a theoretical problem it has a long history starting with the pioneering work of Kac (1943) and Rice's (1944-45). Through further theoretical development the generalized Rice formula approach has been adopted to derive long-run distributions of characteristics taken at random events. While the approach, in principle, is applicable in a quite general setup, even for the Gaussian models there are some critical computational bottle necks. They arise from the integrals formulas which involve high dimensional multivariate joint distributions that can be either close to singular or with a complex correlation structure in high dimensions.

The problems can be best visualized for random surfaces and the Gaussian sea surface model can serve as a classical example of empirical context. For example, one can ask about statistical distribution of wave sizes, in particular, how distributed large waves are or how steep they are. Dynamical evolution of the shapes and its statistical properties can be analyzed through random velocities defined on the moving random surface. A method of measuring three-dimensional spatial wave size can be also introduced and statistical distributions of the size characteristics can be derived for Gaussian sea surfaces. All these distributions lead to integral formulas involving continuous random fields and their computation requires some efficient computational approach.

Things complicate computationally further if the underlying model is non-Gaussian. An example of a non-Gaussian model is moving average process driven by a non-Gaussian noise. The Slepian models that describe the distributional form of a stochastic process observed at level crossings is a convenient way of representing the distributions at random events. The Slepian model can be used for efficient simulations of the behavior of a random process sampled at level-crossings. However, the effective utilization of such a model requires producing samples which, especially in the non-Gaussian case, becomes a non-trivial computational problem. In some important cases, it can be solved by clever conditional sampling in the spirit of Gibbs sampler. The practical application can be seen in analysis of mechanical vehicle responses to a stochastic road surface.

References

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