The Rational SPDE Approach for Gaussian Random Fields With General Smoothness

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Abstract

A popular approach for modeling and inference in spatial statistics is to represent Gaussian random fields as solutions to stochastic partial differential equations (SPDEs) of the form $L^{\beta}u = \mathcal{W}$, where $\mathcal{W}$ is Gaussian white noise, L is a second-order differential operator, and $\beta > 0$ is a parameter that determines the smoothness of u (Lindgren et al. 2011). However, this approach has been limited to the case $2\beta \in \mathbb{N}$, which excludes several important covariance models and makes it necessary to keep β fixed during inference.

We introduce a new method, the rational SPDE approach, which is applicable for any $\beta > 0$ and therefore remedies the mentioned limitation. The presented scheme combines a finite element discretization in space with a rational approximation of the function $x^{-\beta}$ to approximate u. For the resulting approximation, an explicit rate of strong convergence to u is derived and we show that the method has the same computational benefits as in the restricted case $2\beta \in \mathbb{N}$ when used for statistical inference and prediction. Several numerical experiments are performed to illustrate the accuracy of the method, and to show how it can be used for likelihood-based inference for all model parameters including β.

References

1. F. Lindgren and H. Rue and J. Lindström . An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach (with discussion). J. Roy. Statist. Soc. Ser. B Stat. Methodol. 73 (2011) 423–498.