# On Multi-channel Stochastic Networks With Markov Control

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## Abstract

Development and research of analysis techniques for queueing networks with controlled input flow are actual directions of development of the queueing networks theory.

The main model we consider is a queueing network consisting of $r$ service nodes. Each node is a queueing system and it consists of infinite number of servers. Therefore, if a customer arrives at such a system, then it begins processing immediately. Input flow arriving at the network is controlled by a Markov process. We define a service process in the network as an $r$-dimensional stochastic process $Q(t) = (Q_1(t), ..., Q_r(t))'$, $t \geq 0$, where $Q_i(t)$, $i = 1, 2, \ldots, r$, is the number of customers at the $i$-th node at instant $t$.

We study such networks in two cases. Firstly, we consider one-dimensional case, where the network has the only service node. It is assumed that the instants of customers' arrivals to the system are the same as jump instants of a homogeneous continuous-time Markov chain with a finite set of states. A customer arrived to the system immediately begins to be served anywhere on a free server. The service time is distributed exponentially. In this case a generating function of the stationary distribution for the process $Q(t)$ is obtained. The form of the generating function is a matrix version of the Takacs formula.

Further, the network with $r > 1$ service nodes is studied. A common input flow of customers arrives at servicing nodes. This flow is controlled by a Markov chain $\eta(t)$ according to the following algorithm. As before, the instants of customers arrivals are the same as jump instants $t_n$, $n = 1, 2, ...$, of the chain $\eta(t)$. If the chain $\eta(t)$ jumps into state $i$ at the instant $t_n$, the customer numbered $n$ arrives for service into the $i$th node. Note, that the number of states for controlling Markov chain $\eta(t)$ coincides with the number of network nodes. At the node the customer occupies a free server for the time distributed exponentially with parameter $\mu_i$. After service in the $i$th node the customer is transferred to the $j$th node with probability $p_{ij}$, $j = 1, 2, \ldots, r$, or leaves the network with probability $p_{ir+1} = 1 - \sum\limits_{j=1}^{r} p_{ij}$. For a multivariate service process the condition of a stationary regime existence and a correlation matrix are found.

Finally, the stochastic network with controlled input flow is considered in heavy traffic. It is proved, that under certain heavy traffic conditions on the network parameters, the service process converges in the uniform topology to a Gaussian process. Correlation characteristics of the limit process are written via the network parameters.

## References

1. V.V. Anisimov and E.A. Lebedev. Stochastic Queueing Networks. Markov Models. Kiev Univ., Kiev, 1992.