Geometric Multigrid Methods for Darcy–Forchheimer Flow in Fractured Porous Media

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Abstract

In this work, we consider single-phase flow models in fractured porous media. Let us define an n-dimensional convex domain $\Omega \subset \mathbb{R}^n$ with boundary $\partial\Omega$, for $n = 2$ or 3, and an $(n-1)$-dimensional subset $\gamma \subset \Omega$ with boundary $\partial\gamma$ (i.e., the fracture) that divides Ω into two subdomains Ω_1 and Ω_2 with boundaries $\partial\Omega_1$ and $\partial\Omega_2$, respectively (i.e., the porous medium matrix). Following [1], we consider linear Darcy flow in the subdomains,

$$\mathbf{u}_i = -K_i \nabla p_i, \qquad \operatorname{div} \mathbf{u}_i = q_i, \qquad \text{in } \Omega_i, \text{ for } i = 1, 2,$$

with $p_i = g_i$ on $\partial\Omega_i$, for $i = 1, 2$, and nonlinear Darcy–Forchheimer flow in the fracture,

$$(1 + b\,|\mathbf{u}_f|)\,\mathbf{u}_{f,\tau} = -K_{f,\tau}\, d\, \nabla_\tau p_f, \qquad \operatorname{div}_\tau \mathbf{u}_{f,\tau} = q_f + (\mathbf{u}_1 \cdot \mathbf{n} - \mathbf{u}_2 \cdot \mathbf{n}), \qquad \text{in } \gamma,$$

with $p_f = g_f$ on $\partial\gamma$, together with the interface condition

$$\alpha_f\, p_i = \alpha_f\, p_f + (-1)^{i+1}(\xi\, \mathbf{u}_i \cdot \mathbf{n} + (1-\xi)\, \mathbf{u}_{i+1} \cdot \mathbf{n}), \qquad \text{in } \gamma, \text{ for } i = 1, 2,$$

where $\alpha_f = \frac{2K_{f,n}}{d}$, $\xi \in \left(\frac{1}{2}, 1\right]$, and $i + 1 = 1$ if $i = 2$. Here, p_j is the fluid pressure, $\mathbf{u}_j$ is the Darcy velocity, K_j is a diagonal permeability tensor, and q_j is a source term, with $j = 1, 2, f$ corresponding to Ω_1, Ω_2 and γ, respectively. Further, $\mathbf{u}_{f,\tau}$ denotes the tangential component of $\mathbf{u}_f$, and $K_{f,\tau}$ and $K_{f,n}$ are the tangential and normal components of K_f. Finally, b is the Forchheimer coefficient, d is the fracture width, and $\mathbf{n}$ is the unit normal vector on γ directed outward from Ω_1. Note that ∇_τ and div_τ represent the tangential gradient and divergence operators, respectively.

We discretize the preceding equations using a two-point flux approximation scheme that takes into account the mixed-dimensional nature of the problem at hand. Monolithic geometric multigrid methods are then proposed for solving the resulting system of algebraic equations. Numerical experiments illustrating the behaviour of the algorithms are shown.

References

1. N. Frih; J.E. Roberts; A. Saada. Modeling fractures as interfaces: a model for Forchheimer fractures. Comput. Geosci. 12 (2008) 91–104.