Finite Element Approximation of an Elliptic Problem With a Nonlinear Newton Boundary Condition

Miloslav Feistauer, Ondrej Bartos, Filip Roskovec
Department of Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, 18675 Prague, Czech Republic
feist@karlin.mff.cuni.cz, ondra.bartosh@seznam.cz, roskovec@gmail.com

Abstract

Many engineering problems can be described using elliptic partial differential equations equipped with nonlinear Newton boundary conditions, e.g. electrolysis of aluminium or radiation heat transfer problem. We look at a problem which contains (possibly non-integer) powers in its boundary condition in a two-dimensional polygonal domain. The exact solution loses regularity near boundary vertices and to some extent also near boundary edges. When this problem is discretized to seek a piecewise polynomial approximate solution and solved using a finite element method (FEM) this loss of regularity limits the theoretically expected order of convergence of the approximate solution to the exact solution [1]. Additionally, the order of convergence should be divided by the power appearing in the nonlinear boundary condition [2]. In practise, FEM often converges with such a rate as if there was no nonlinearity on the boundary. We attempt to explain this behaviour, whether it is the same if the error is measured in different norms of Sobolev spaces and how this changes when the exact solution is zero on a large part of the boundary. An interesting phenomenon is that the derivatives of the approximate solution can converge faster to the exact solution than the function values themselves.

References

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