Polyharmonic Splines Generated by Multivariate Smooth Interpolation

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Abstract

Splines can be derived in two different ways: the algebraic one (where splines are understood as functions defined piecewise with smoothness conditions at points of change of the function definition) and the variational one (where splines are obtained by minimization of a quadratic functional with constraints), see, e.g., Segeth (2018) for the case of cubic 1D spline.

We show that the general variational approach called smooth interpolation (first introduced by Talmi and Gilat (1977)) can be extended from 1D interpolation (odd degree polynomial splines) to the multivariate case and the order of the spline can be extended as compared with Mitáš and Mitášová (1988).

To this end, we choose the system of functions $\exp(-ik \cdot x)$ for the basis of the space where we minimize functionals and measure the smoothness of the result. The general form of the interpolation formula is then the linear combination of the values of some radial basis functions and low-order polynomials (called the trend in Mitáš and Mitášová (1988)) at nodes, see Segeth (2018). The dimension considered in this contribution is 1, 2, and 3.

We also mention the problem of smooth curve fitting (data smoothing) and present a simple numerical example. Smooth approximation in 2D and 3D can be a very useful tool e.g. in computer aided geometric design or geographic information systems.

References

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