

A Brief Reflection About Iterative Sentences and Arithmetic

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Abstract

I have taught mathematics with computers to students of the School of Education for 32 years within the frame of different subjects and using different languages and hardware. Now I use Scratch, Maple and GeoGebra. The goals of these subjects are to show how the computer can help in the learning process in Primary and/or Secondary Education and to clarify and strengthen the concepts of our students. Mastering a computational language or package is not a goal. And I believe it is advisable to always introduce, when possible, mathematical or computational ideas using examples well known by the students (in another context). For instance, a procedure can be compared with the step by step explanation of a tennis shot.

Let us focus on a very specific problem: implementing arithmetic operators as nontrivial examples of iterative sentences (depending on the level of the audience, I later introduce them in a recursive way or not). This is not Peano's arithmetic, but it is also a constructive approach. This year we were working with Scratch 2 [1], and I proceeded as usual:

- i) I explained how we calculate by hand a power which exponent is a positive integer and pushed the students to rediscover the process underneath (in order to implement it).
- ii) Exercise: do the same for multiplication (from addition),
- iii) Exercise: do the same for addition (from the elementary operation "add 1" -successor)
- iv) Exercise: implement the factorial function.

My experience is that this top to down order is the best possible one. It is clear that iv) goes after i), ii), iii) because you have to multiply by a number that changes. But why i), ii) and iii) are best introduced in that order, although they seem of increasing "complexity"? (really, the algorithms are almost identical). In my opinion, the reasons are:

- i) We don't memorize "power tables", and what we do by hand to calculate a power which exponent is a positive integer is exactly the algorithm proposed.
- ii) Although we memorize multiplication tables, we sometimes use this procedure to mentally calculate the result in some cases like, e.g., $3 \cdot 1250$, so this process is somehow "fresh".
- iii) This is the process carried out to sum by a kid that can count but still don't know how to sum ("count with the fingers"). For instance, $3 + 4$ would correspond to count 3 fingers plus four times one more finger. As we haven't done this for a long time, it seems more difficult. Another reason is that "successor" is not recognized as an operation.

This is just a reflection after a long teaching experience, but I think it is a curious hypothesis.

References

1. .ANONYMOUS . Scratch 2 web page. <https://scratch.mit.edu/>.