Robust Discrete Laplacians

Antonio DiCarlo
CECAM-IT-SIMUL Node
adicarlo@mac.com

Abstract

Computing an approximate solution to the Poisson problem on an n-dimensional Riemannian manifold (with boundary) is at the heart of a host of numerical applications. Thus, the fact that no satisfactory discrete approximation to the Laplace-de Rham operators Δ_k is generally available for any $k \leq n$ is highly annoying. In a nice paper published ten years ago, Wardetzy et al. [1] proved that, on a general mesh, it is impossible to construct a discrete Laplacian sharing four important structural properties possessed by the differential Laplace-de Rham operator Δ_0 acting on 0-forms (i. e., scalar fields), namely: (i) linearity, (ii) symmetry, (iii) positivity, and (iv) locality. Of course, any decent approximation to Δ_0 will asymptotically recover these properties in the limit of infinite mesh refinement. However, one would like to preserve (most of) them on any coarse mesh.

Building on ideas introduced by DiCarlo et al. [2], I argue that the one requirement to be dropped—or better, relaxed—is locality, since this renunciation allows us to produce very robust approximations to Δ_k for all k's, tolerant of poor-quality meshes. This robustness is especially beneficial when coping with solid models extracted from big geometric data [3] or obtained by merging two or more cellular complexes [4]: even if topologically correct, these meshes typically have inferior metric properties.

Interestingly, the approach in [2] combines well also with the idea of constructing localised bases by spectral decomposition of a modified Laplacian operator, recently put forward by Melzi et al. [5].

References

1. M. Wardetzky and S. Mathur and F. Kaelberer and E. Grinspun. Discrete Laplace operators: No free lunch. Symposium on Geometry Processing. Barcelona, Spain, July 4–6, 2007, Eurographics Association (2007), 33–37, ISBN: 978-3-905673-46-3.
2. A. DiCarlo and F. Milicchio and A. Paoluzzi and V. Shapiro. Discrete physics using metrized chains. Symposium on Solid and Physical Modeling. San Francisco, CA, October 5–8, 2009, ACM (2009), 135–145, ISBN/ISSN: 978-1-60558-711-0.
3. A. Paoluzzi and A. DiCarlo and F. Furiani and M. Jiřík. CAD models from medical images using LAR. Comput. Aided Des. Appl. **13**:6, 747–759 (2016).
4. A. Paoluzzi and V. Shapiro and A. DiCarlo. Regularized arrangements of cellular complexes. ArXiv:1704.00142v4 [cs.CG] 25 Aug 2017.
5. S. Melzi and E. Rodolà and U. Castellani and M.M. Bronstein. Localized manifold harmonics for spectral shape analysis. ArXiv:1707.02596v2 [cs.GR] 2 Nov 2017.