# Optimization of Error Indicators for SOLD Methods

Petr Lukas
Charles University in Prague
lukas@karlin.mff.cuni.cz

Pavel Solin
University of Nevada, Reno
solin@unr.edu

## Abstract

In the talk we consider the numerical solution of the scalar convection–diffusion–reaction equation

$$-\varepsilon\,\Delta u + \mathbf{b}\cdot\nabla u + c\,u = f \ \text{ in } \Omega, \qquad u = u_b \ \text{ on } \Gamma^D, \qquad \varepsilon\,\frac{\partial u}{\partial \mathbf{n}} = g \ \text{ on } \Gamma^N. \tag{1}$$

We present new results of an adaptive technique in finite element method based on minimizing a functional called error indicator $I_h : W_h \to \mathbb{R}$, where $W_h$ is a FE space for the discrete solution $w_h$ of (1). The simplest form of such an indicator is

$$I_h(w_h) = \sum_{K\in\mathcal{T}_h,\overline{K}\cap\partial\Omega=\emptyset} h_K^2 \ \| - \varepsilon\Delta w_h + \boldsymbol{b}\cdot\nabla w_h + c w_h - f\|_{0,K}^2 \qquad \forall w_h \in W_h, \tag{2}$$

where we have used the usual notation from the article of V. John, P. Knobloch, S. B. Savescu [1]. It is possible to enrich this indicator by other terms, which favour a less smeared solution to a diffuse one. One example of such an added term is $\|\phi(|\boldsymbol{b}^\perp\cdot\nabla w_h|)\|_{0,1,K}$, where $\phi$ is a function like square root. We can also bound the residue term in (2) from above to obtain a physical solution for examples with non-steep Dirichlet boundary conditions. The suitability of added terms depends on the problem we solve.

The parameter we are changing in the optimization procedure is currently the parameter $\tau$ from SUPG (SDFEM) method and the parameter called $\tilde{\epsilon}$ in [2] for the SOLD method which adds diffusion in the crosswind direction (page 2205). We use several different FE spaces for both parameters, see [3].

## References

1. V. John and P. Knobloch and S. B. Savescu. A posteriori optimization of parameters in stabilized methods for convection-diffusion problems - Part I. Comput. Methods Appl. Mech. Engrg. 200 (2011), 2916-2929.
2. V. John and P. Knobloch. On spurious oscillations at layers diminishing (SOLD) methods for convection-diffusion equations: Part I - A review. Comput. Methods Appl. Mech. Engrg. 196 (2007), 2197-2215.
3. P. Lukas and P. Knobloch. Adaptive techniques in SOLD methods. Applied Mathematics and Computation 319 (2018), 24-30.