On the Robustness of Hierarchical Bayesian Models for Uncertainty Quantification in Inverse Problems

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Abstract

The two primary challenges with classical variational approaches for linear inverse problems in modern scientific modeling and computing are the manual selection of regularization parameters and the inability to directly model and quantify uncertainties. In order to address the issue of uncertainty quantification, the last decade has seen a transformational shift to Bayesian models for inverse problems and the development of Markov Chain Monte Carlo (MCMC) or other sampling approaches to computing posterior distributions of "solutions," in place of single, approximately optimal answers. Straightforward Bayesian models still require scaling parameters, as a measure of how much one trusts their data vs. their prior, retaining an analogy to the classical problem of regularization parameter selection. To deal with both problems of parameter selection and uncertainty quantification, there has been a move toward hierarchical Bayesian models, in which one puts a prior distribution on the scale parameters then selects the so-called hyperprior parameters for the prior distributions [1,2,3]. In this setting, one replaces a few, presumed highly sensitive, scale parameters with a larger number of, presumed highly insensitive, hyperprior parameters. In this work we show that for several linear inverse problems, the computation of the mean of the posterior for hierarchical Bayesian models, as computed with Gibbs samplers or other similar MCMC methods, is highly robust to the manual selection of hyperprior parameters, but that the computation of the variances of the model parameters with hierarchical models is very sensitive to the hyperprior parameter selection. This suggests that hierarchical models are an excellent approach to ameliorating the classical problem of regularization parameter selection, but that uncertainty quantification with such models is, at least for some common problems, still tenuous.

References

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