Finite Difference Solutions of 1D Magnetohydrodynamic Channel Flow With Slipping Walls

Sinem Arslan, Münevver Tezer-Sezgin
Middle East Technical University, Department of Mathematics
arsinem@metu.edu.tr, munt@metu.edu.tr

Abstract

In this study, the fully developed magnetohydrodynamic (MHD) flow is considered in a pipe along with the z-axis under an external magnetic field which is perpendicular to the pipe. So, the relevant variables, the velocity u and the induced magnetic field b depend on the plane coordinates x and y. When the flow is considered between two parallel plates (Hartmann flow) the external magnetic field is perpendicular to the two channel walls and the lateral channel walls are at infinity. Now, the variations of the velocity and the induced magnetic field are only with respect to the coordinate y between the plates, [1]. The finite difference method (FDM) is used to solve the governing equations with different type of boundary conditions such as the slip boundary condition for the velocity u and insulated and/or conducting end conditions for the induced magnetic field b. The numerical results obtained from FDM discretized equations have been compared with the exact solution whenever it exists (i.e. the no-slip walls) for the 1D MHD flow between parallel plates. The velocity and the induced magnetic field are obtained at the mesh points and simulated for each case of boundary conditions. It is observed from the profiles of the velocity that the velocity magnitude decreases as the Hartmann number Ha increases, which is the well-known flattening tendency of MHD flow. Also, it is seen that boundary layers for both the velocity and the induced magnetic field are formed near the plates when Ha increases. Further, the effects of the increase in both the conductivity parameter c and the slip length α on the flow and on the induced magnetic field are shown for several values of Ha. The increases in the slip length and the conductivity parameter, increase the magnitudes of the velocity and the induced current, respectively, which are both weakened for large values of Ha. The volumetric flow rate decreases with an increase in the wall conductivity whereas it increases with an increase in the slip length. Thus, the FDM which is simple to implement, enables one to depict the effects of these type of boundary conditions on the behaviour of both the velocity and the induced magnetic field at a small expense.

References

1. Muller U. and Buhler L. Magnetohydrodynamics in Channels and Containers. Springer, New York, 2001.