Optimal Control for the MHD Flow and Heat Transfer With Variable Viscosity in a Square Duct

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Abstract

The direct and optimal control solution of the laminar, fully developed, steady MHD flow of an incompressible, electrically conducting fluid in a duct is considered together with the heat transfer. The flow is driven by a constant pressure gradient and an external uniform magnetic field. The fluid viscosity is either temperature dependent, varying exponentially, or it depends on the flow in the case of power law fluid; and the viscous and Joule dissipations are taken into consideration. The coupled nonlinear set of momentum and energy equations are solved by using Finite Element Method with the implementation of the Newton's method for nonlinearity. In this respect, direct FEM solutions are obtained for various values of the problem parameters to ensure the sound structure of the underlying scheme. The FEM results obtained in this study are not only in good agreement with, but also extends, the results in [1]. The aim of this study is to investigate the problem of controlling the steady flow by using the physically significant parameters of the problem as control variables: Hartmann number (Ha), Brinkmann number (Br), Hall parameter (m) and viscosity parameter (B) in the case of temperature dependent viscosity. For the case of power law fluid the control parameters are Ha, Br and the flow index (n). The control problem is solved by the *discretize-then-optimize* approach [2] with a gradient based algorithm. Starting with an initial estimate the optimization loop to calculate new estimates for optimal solution repeated until the norm of the gradient of the reduced cost function is less than a predefined tolerance. Control variables are considered as single and pairwise as well. It is observed that controls with multiple control variables require relatively more number of iterations than the one with a single control parameter. The most costly one is observed as the case with the pair (m, B) since they have contrary effects on the fluid for the temperature dependent viscosity case. Numerical results ensure that the proposed control approach is effective at driving the flow to prescribed velocity profiles as well as isolines.

References

1. M. E. Sayed-Ahmed. Numerical solution of power law fluids flow and heat transfer with a magnetic field in a rectangular duct. International Communications in Heat and Mass Transfer, 33 (2006), 1165-1176.
2. M. Hinze and R. Pinnau and M. Ulbrich and S. Ulbrich. Optimization with PDE Constraints. Springer, 2009.